

BAYESIAN HIERARCHICAL MODELS FOR UNCERTAINTY QUANTIFICATION IN STRUCTURAL DYNAMICS

GUILLEM C. BALLESTEROS¹, PANOS ANGELIKOPOULOS¹, COSTAS
PAPADIMITRIOU² and PETROS KOUMOUTSAKOS¹

²*Computational Science and Engineering Laboratory, ETH Zurich, CH-8092 Zurich,
Switzerland.*

E-mail: guillemb@ethz.ch, panagiotis.angelikopoulos@mavt.ethz.ch, petros@ethz.ch

¹*Department of Mechanical Engineering, University of Thessaly, GR-38334 Volos, Greece.*

E-mail: costasp@uth.gr

Identical structural components used in mechanical systems often manifest differences in their dynamics properties, such as modal frequencies and modeshapes, due to manufacturing, material, geometric, boundary, contact and assembly variabilities. Experimental data from a number of identical structural components of mechanical systems can be used to quantify the uncertainties in the mechanical properties of these components and propagate these uncertainties in simulations for robust response and reliability predictions. The uncertain properties may include the modulus of elasticity, stiffness of subcomponents, boundary conditions, spatial variations of mechanical properties, friction coefficients at contact surfaces, parameters associated with impact models, etc. The uncertainties in the values of these parameters can be quantified using probability distributions, where the parameters of the probability distributions, often called hyper-parameters, are considered unknown to be computed from the experimental data. The aforementioned models constitute hierarchical models and the Bayesian inference framework can be used to quantify uncertainties based on the experimental data and propagate these uncertainties in predictions of important quantities of interest.

In this work, the Bayesian framework for inference in hierarchical models is first reviewed. Parameterized Gaussian models of uncertainties are introduced to model uncertainties. The joint posterior distribution of the structural model parameters and hyper-parameters is formulated. The space of uncertain parameters (physical parameters and hyper-parameters) grows linearly with the number of tested structural components. The Bayesian tools for estimating the posterior distribution of structural model parameters and the hyper-parameters, as well as performing robust prediction analyses are Laplace methods of asymptotic approximation and more accurate stochastic simulation algorithms (e.g. Transitional MCMC [1]). These tools involve solving optimization problems, generating samples for tracing and then populating the important uncertainty region in the parameter space, as well as evaluating integrals over high-dimensional spaces of the uncertain model parameters. They require a moderate to very large number of repeated system analyses to be performed over the large-dimensional space of uncertain parameters.

Two novel approaches are presented and their efficiency is evaluated. The first one has to do with the approximation of the posterior distribution of the hyper-parameters. This distribution is formulated as the product of as many multi-dimensional integrals as the number of structural components tested, while the dimension in each integral is the same as the number of physical uncertain model parameters. Laplace methods of asymptotic approximations are shown to be computationally efficient to approximate these integrals. The second method is related to

generating samples from the posterior distribution in the large dimensional parameter space. To avoid very small acceptance rate manifested in conventional Metropolis-Hasting (MH) sampling algorithms, a Blocked Gibbs sampler is proposed to generate samples simultaneously along the different uncertain parameter subspaces associated with the different number of structural component tested. High performance computing techniques [3] can be integrated with the Bayesian framework to exploit the parallelized capabilities of the proposed framework and efficiently handle the large number of repeated computations by distributing them in available multi-core CPUs. Drastic reductions in computational effort are achieved.

Theoretical and computational developments of the framework are illustrated using simulated and experiment data for modal frequencies and modeshapes obtained from a number of identical beam structures tested under slightly different configuration conditions. It is demonstrated that the proposed Bayesian computational tools are quite general and efficient for handling the model parameter and hyper-parameter uncertainties for uncertainty quantification and propagation in structural dynamics.

Keywords: Bayesian inference, hierarchical models, asymptotic approximations, MCMC, structural dynamics.

Acknowledgements

This research has been implemented under the “ARISTEIA” Action of the “Operational Programme Education and Lifelong Learning” and was co-funded by the European Social Fund (ESF) and Greek National Resources.

References

1. J. Ching and Y.C. Chen, *ASCE Journal of Engineering Mechanics*, 133, 816–832, 2007.
2. S.K. Au and J.L. Beck, *Probabilistic Engineering Mechanics*, 16, 263-277, 2001.
3. P. Angelikopoulos, C. Papadimitriou and P. Koumoutsakos, *The Journal of Chemical Physics*, 137(14), 2012.